You must show steps to receive credit.

1. Find the equation of the plane containing the points $P(1,-1,2)$, $Q(0,1,1)$, and $R(2,0,1)$.

2. For all real $k$, find the traces in the planes $x = k$, $y = k$, and $z = k$ of the surface $x^2 + y^2 - z^2 = 1$. Identify the surface.

3. Find parametric equations for the tangent line to the curve with the parametric equations
   $$x = t^5, y = t^4, z = t^3$$
   at the point $(1,1,1)$.

4. Find the length of the curve $r(t) = i + t^2j + t^3k$, $0 \leq t \leq 1$.

5. (See separate sheet.) Match each of the functions to its graph and its contour map. In each graph the bottom right-hand edge is the $x$-axis. For each of (a)-(f), your answer should include one of (a)-(f) and one of (i)-(vi).
   
   (a) $z = x^2$
   (b) $z = -xye^{-x^2-y^2}$
   (c) $z = x^2 + y^2$
   (d) $z = \frac{-3y}{x^2 + y^2 + 1}$
   (e) $z = y^3$
   (f) $z = e^x \cos(y)$

6. Determine the set of points at which the function $f$ given below is continuous. Justify your answer.
   $$f(x, y) = \begin{cases} y^4 & \text{if } (x, y) \neq (0,0) \\ x^4 + 3y^4 & \text{if } (x, y) = (0,0) \end{cases}$$

7. Let $f(x, y) = x \ln(x^2 + y^2)$.
   
   (a) Find the first partial derivatives of the function $f$.
   (b) Find the second partial derivative $f_{yy}(x, y)$.

8. Find the equation of the tangent plane to the surface $z = 2x^2 + y^2$ at the point $(1,1,3)$.

9. If $z = f(x, y)$, where $f$ is differentiable, $x = g(t)$, $y = h(t)$, $g(3) = 2$, $g'(3) = 5$, $h(3) = 7$, $h'(3) = -4$, $f_x(2,7) = 6$ and $f_y(2,7) = -8$, find $dz/dt$ when $t = 3$.

10. Let $f(x, y, z) = y^2e^{xz^3}$.
    
    (a) Find the directional derivative $D_vf(0,1,1)$ in the direction of $v = i + j + k$ at the point $(0,1,1)$.
    (b) Find the maximum rate of change of $f$ at the point $(0,1,1)$ and
c    (c) the direction in which it occurs.
Figure 1: Graphs

Figure 2: Contour Maps
10.1 \( \mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} T & S & R \\ 1 & 1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = -7 - 2z - 3z^2 \\
\Rightarrow x - 1 + 2(y + 1) + 3(z - 2) = 0 \quad \Rightarrow x + 2y + 3z = 5 \\
\) 

10.2 \( x = k \quad \Rightarrow \quad y + z = 1 - k^2 \) hyperbola for all \( k \)  
\( y = k \quad \Rightarrow \quad x + z = 1 - k^2 \) hyperbola for all \( k \)  
\( z = k \quad \Rightarrow \quad x^2 + y^2 = 1 + k^2 \) circle for all \( k \)  
Circular hyperboloid \( \rightarrow \) hyperboloid of one sheet

10.3 \( \mathbf{v} = \mathbf{r}'(t) = < 5t, 4t^2, 3t^3 > \) with \( t = 1 \).  
\( \mathbf{v} = < 5, 4, 3 > \) parametric eq of tangent line: \( x = 1 + 5t, y = 1 + 4t, z = 1 + 3t \)

10.4 \( L = \int_1^2 \sqrt{1 + (5t^2 + 4t^4 + 9t^6) dt} = \int_1^2 \sqrt{1 + 4t^4} dt = \frac{3}{10} (4 + 9t^6)^{3/2} \left| _1^2 \right. \)
\( L = \frac{1}{2} \sqrt{12} \left( \frac{3}{10} (13 - 8) \right) = \frac{1}{10} \sqrt{11} \)

12.5 \( a) \quad \frac{\partial}{\partial x} \left( \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) \)  
\( b) \quad \frac{\partial}{\partial y} \left( \frac{\partial}{\partial z} + \frac{\partial}{\partial x} \right) \)  
\( c) \quad \frac{\partial}{\partial z} \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \)

8.6 \( f \) is continuous at all points \((x, y) \neq (0, 0)\). This is because \( f \) is a rational function defined for all points \((x, y) \neq (0, 0)\).

Let \( y = 0 \), \( f(x, 0) = 0, x \neq 0 \). Let \( x \to 0 \), \( f(0, y) = 0 \), \( y \to 0 \).

\( f \) does not exist. \( f \) is not continuous at \((0, 0)\).

8.10 \( \frac{\partial^2}{\partial x^2} = 4x \)  
\( \frac{\partial^2}{\partial y^2} = 2y \) at \((1, 1)\) equal \( 4 \) \( \neq \) \( 2 \) resp.  
\( \frac{\partial}{\partial z} = 4(x - 1) + 2(y - 1) \)  
\( \frac{\partial}{\partial z} = 4x + 2y - z \)

8.9 \( \mathbf{z} = \mathbf{f}(x, y) \), \( x = g(t), y = h(t) \)  
\( \frac{dx}{dt} = f_x(x, y)g'(t) + f_y(x, y)h'(t) \)
\( t = 3 \Rightarrow x = g(3) = 2, y = h(0) = 7 \)  
\( \frac{dx}{dt} \bigg|_{t=3} = f_x(x, y)g'(3) + f_y(x, y)h'(3) \)
\( = 6 \cdot 5 + (-8) \cdot (-4) = 30 + 32 = 62 \)

12. \( \nabla f(x, y, z) = < z^2 y^3 z^2 z^2, y^3 z^2 y^3 z^2 > \)
\( \nabla f(0, 1, 1) = < 1, 2, 0 > \) \( \mathbf{u} = \sqrt{5} \left( \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, \frac{0}{\sqrt{5}} \right) < 1, 1, 1 > \)

(a) \( D_x f(0, 1, 1) = < 1, 2, 0 > \cdot \mathbf{u} = \sqrt{5} \)

(b) \( \) max rate of change of \( f = \sqrt{5} \)

(c) \( \) direction = \( < 1, 1, 1 > \)